

Deriving the Equation for Tangent Lines

The work below shows how I derived the functions that generate the tangent lines for the programs `sinTangents.ch`, `prabTangents.ch`, `tanTangents.ch`, `expTangents.ch`, and `lnTangents.ch`. Each of these programs generate several images that can be merged together to create an animation for the tangent lines of these functions. All of the code and animations can be found at www.mrdagler.com/tangent_lines.html

- **Tangent lines of $f(x) = \sin(x)$**

Find the equation of the tangent lines for $f(x) = \sin(x)$ at the point $(i, \sin(i))$.

Since $f'(x) = \cos(x)$, we know that the slope of the tangent lines is given by $m = \cos(i)$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - \sin(i) &= \cos(i)(x - i) \\y - \sin(i) &= \cos(i)x - \cos(i)i\end{aligned}$$

$$\boxed{y = \cos(i)x - \cos(i)i + \sin(i)}$$

- **Tangent lines of $f(x) = x^2$**

Find the equation of the tangent lines for $f(x) = x^2$ at the point (i, i^2) .

Since $f'(x) = 2x$, we know that the slope of the tangent lines is given by $m = 2i$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - i^2 &= 2i(x - i) \\y - i^2 &= 2ix - 2i^2\end{aligned}$$

$$\boxed{y = 2ix - i^2}$$

- **Tangent lines of $f(x) = \tan(x)$**

Find the equation of the tangent lines for $f(x) = \tan(x)$ at the point $(i, \tan(i))$.

Since $f'(x) = \sec^2(x)$, we know that the slope of the tangent lines is given by $m = \sec^2(i)$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - \tan(i) &= \sec^2(i)(x - i) \\y - \tan(i) &= \sec^2(i)x - \sec^2(i)i\end{aligned}$$

$$\boxed{y = \sec^2(i)x - \sec^2(i)i + \tan(i)}$$

or

$$\boxed{y = \frac{x}{\cos^2(i)} - \frac{i}{\cos^2(i)} + \tan(i)}$$

• **Tangent lines of $f(x) = e^x$**

Find the equation of the tangent lines for $f(x) = e^x$ at the point (i, e^i) .

Since $f'(x) = e^x$, we know that the slope of the tangent lines is given by $m = e^i$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - e^i &= e^i(x - i) \\y - e^i &= e^i x - e^i i\end{aligned}$$

$$\boxed{y = e^i x - e^i i + e^i}$$

• **Tangent lines of $f(x) = \ln(x)$**

Find the equation of the tangent lines for $f(x) = \ln(x)$ at the point $(i, \ln(i))$.

Since $f'(x) = \frac{1}{x}$, we know that the slope of the tangent lines is given by $m = \frac{1}{i}$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - \ln(i) &= \frac{1}{i}(x - i) \\y - \ln(i) &= \frac{x}{i} - 1\end{aligned}$$

$$\boxed{y = \frac{x}{i} - 1 + \ln(i)}$$